問. (2) $\log 1 = 0$, $\log e = 1$ を示せ.

$y = \log x \overset{def.}{\iff} x = e^y$ より

$y_1 = \log 1 \iff 1 = e^{y_1}$ $\therefore y_1 = 0$

$y_2 = \log e \iff e = e^{y_2}$ $\therefore y_2 = 1$

(e^xの単調性を用いている. 指数関数は $x \neq y \iff e^x \neq e^y$)

問. 証明せよ. (授業でやった)

(1) $a^{x+y} = a^x a^y$ (2) $a^0 = 1$ (3) $a^1 = a$ (4) $a^{-x} = \frac{1}{a^x}$

(2) $a^0 = e^{(\log a)\cdot 0} = e^0 = 1$

(3) $a^1 = e^{(\log a)\cdot 1} = e^{\log a} = a$

(4) $a^{x-x} = a^x (a^{-x})$ ($\because$ (1))

$= a^0$

$= 1$ より $a^{-x} = \frac{1}{a^x}$

問. (1) $f(x) = C \in \mathbb{R} \longrightarrow f'(x) = 0$

(2) $f(x) = x^n$ $(n = 1, 2, 3, \cdots) \longrightarrow f'(x) = nx^{n-1}$

(3) $f(x) = x^{-n}$ $(n = 1, 2, 3 \cdots) \longrightarrow f'(x) = (-n)x^{-n-1}$

これら3つは(9)で一般的に示されるが, それぞれ普通に示してみる.

(1) $\frac{d}{dx} C = \lim_{h\to 0} \frac{C - C}{h} = 0$

(2) $\frac{d}{dx} x^n = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$

$= \lim_{h\to 0} \frac{1}{h} \left\{ (x^n + {}_nC_1 h x^{n-1} + {}_nC_2 h^2 x^{n-2} + \cdots + h^n) - x^n \right\}$

$= \lim_{h\to 0} \left({}_nC_1 x^{n-1} + {}_nC_2 h x^{n-2} + \cdots + h^{n-1} \right)$

$= n x^{n-1}$