x方向に垂直な面

$$\therefore \int_{C_x} = -A_x(x,y,z)\cdot(\Delta y)\cdot(\Delta z) + A_x(x+\Delta x,y,z)\cdot(\Delta y)\cdot(\Delta z)$$
$$= \frac{\partial A_x}{\partial x}(\Delta y)(\Delta x)(\Delta z)$$

y方向 $$\int_{C_y} = -A_y(x,y,z)(\Delta x)(\Delta z) + A_y(x,y+\Delta y,z)(\Delta x)(\Delta z)$$
$$= \frac{\partial A_y}{\partial y}(\Delta x)(\Delta y)(\Delta z)$$

z方向 $$\int_{C_z} = -A_z(x,y,z)(\Delta y)(\Delta x) + A_z(x,y,z+\Delta z)(\Delta y)(\Delta x)$$
$$= \frac{\partial A_z}{\partial z}(\Delta z)(\Delta x)(\Delta y)$$

$$\therefore \int_{C上} \mathbf{A}\,d\mathbf{S} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)(\Delta x)(\Delta y)(\Delta z)$$

$$\therefore \frac{1}{V}\int_{C上} \mathbf{A}\cdot d\mathbf{S} = \frac{1}{\cancel{(\Delta x)(\Delta y)(\Delta z)}}\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)\cancel{(\Delta x)(\Delta y)(\Delta z)}$$

$$\lim_{V\to 0}\frac{1}{V}\int_{C上} \mathbf{A}\cdot d\mathbf{S} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

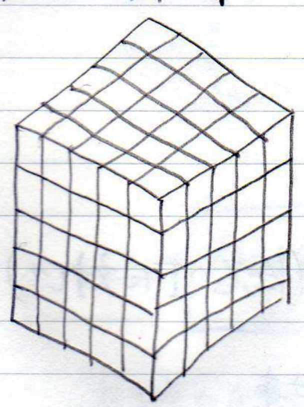
$$\therefore \boxed{\mathrm{div}\,\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

(大きな立体になっても成立)

$$\boxed{\mathrm{div}\,\mathbf{A} = \lim_{V\to 0}\frac{1}{V}\int \mathbf{A}\cdot d\mathbf{S}}$$

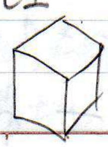
$$\boxed{\mathrm{div}\,\mathbf{E} = \frac{1}{\varepsilon_0}\rho(\mathbf{r})}$$

• 大きな体積 V のとき、



$$\int \mathbf{A}\cdot d\mathbf{S} + \int \mathbf{A}\cdot d\mathbf{S} = \int \mathbf{A}\cdot d\mathbf{S}$$

$$\Longrightarrow \sum \int \mathbf{A}\cdot d\mathbf{S} = \int_{C上} \mathbf{A}\cdot d\mathbf{S}$$



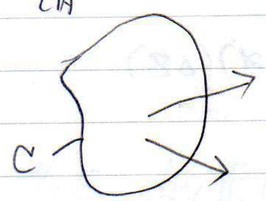
(大)

(復習)

Gaussの法則.

(積分形)

$$\int_{C内} \mathbb{E}\cdot d\mathbb{S} = \frac{1}{\varepsilon_0}(C内の全電気量)$$



$$\boxed{\mathrm{div}\,\mathbb{E} = \frac{1}{\varepsilon_0}\rho(\mathbb{r})}$$ (微分形)

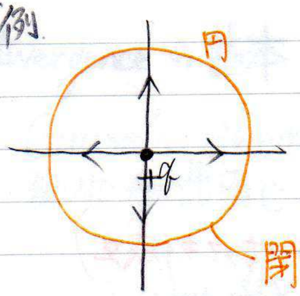
$$\boxed{\mathrm{div}\,\mathbb{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}} \qquad \mathbb{A} = (A_x, A_y, A_z)$$

$$\Updownarrow$$

$$\boxed{\mathrm{div}\,\mathbb{A} = \lim_{V\to 0}\frac{1}{V}\int_{C内}\mathbb{A}\cdot d\mathbb{S}}$$

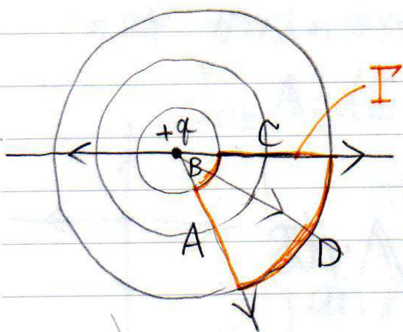
• エネルギー保存則.

(例)



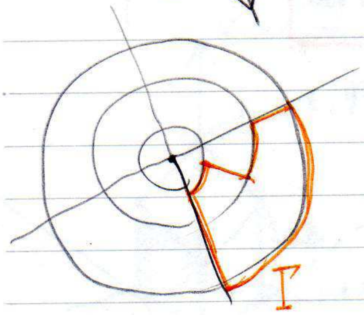
$$\oint_{Γ上} \mathbb{E}\cdot d\mathbb{s} = 0$$

$$d\mathbb{s}\begin{cases} 大きさ：ds\ (微小線要素) \\ 向き：Γの接線方向 \end{cases}$$



$$\oint_{Γ上} \mathbb{E}\cdot d\mathbb{s} = \int_A + \int_B + \int_C + \int_D$$

$$\left\{\begin{array}{l} \int_B = \int_D = 0 \\ \int_A = -\int_D \end{array}\right\} \quad \therefore \int_{Γ上} = 0$$



$$\oint_{Γ上} \mathbb{E}\cdot d\mathbb{s} = 0 \qquad \boxed{\oint_{Γ上} \mathbb{E}\cdot d\mathbb{s} = 0}$$ (任意のΓに対して)

大きさ1の電荷をΓに沿って一周させたときの仕事.

$$W = \oint_{Γ上}(1\times\mathbb{E})\,d\mathbb{s} = 0$$ (エネルギー保存則)