例題 まとめ 後半

例 5-1　$Pa, \forall x(Px \longrightarrow Qx) \vdash Qa$

5-2　$\forall x(Px \wedge Qx) \vdash \forall xPx \wedge \forall xQx$

5-2'　$\forall xPx \wedge \forall xQx \vdash \forall x(Px \wedge Qx)$

5-3　$\forall xPx \vee \forall xQx \vdash \forall x(Px \vee Qx)$　<逆は言えない>

5-4　$\forall x(P \longrightarrow Qx) \vdash P \rightarrow \forall xQx$

5-5　$\forall xPax, \forall x\forall y(Pxy \longrightarrow Qyx) \vdash \forall xQxa$

5-6　$\forall xPx \longrightarrow \forall xQx, \neg Qa \vdash \neg\forall xPx$

6-1　$\forall xPx \vdash \exists xPx$

6-2　$\exists x(Px \wedge Qx) \vdash \exists xPx \wedge \exists xQx$

6-3　$\forall x(Px \longrightarrow Qx) \vdash \exists xPx \longrightarrow \exists xQx$

6-4　$\exists x(Px \wedge Q) \vdash \exists xPx \wedge Q$

6-5　$\exists x\forall yPxy \vdash \forall y\exists xPxy$

6-6　$\neg\exists xPx \vdash \forall x\neg Px$

6-7　$\neg\forall xPx \vdash \exists x\neg Px$

6-8　$\forall x(Px \longrightarrow \neg Qx) \vdash \neg\exists x(Px \wedge Qx)$

5-1

$$\dfrac{\dfrac{Pa \qquad \dfrac{\forall x(Px \to Qx)}{Pa \to Qa}}{Qa}}{}$$

5-2

$$\dfrac{\dfrac{\dfrac{\forall x(Px \wedge Qx)}{Pa \wedge Qa}}{Pa}}{\forall x\, Px} \qquad \dfrac{\dfrac{\forall x(Px \wedge Qx)}{Pa \wedge Qa}}{Qa}}{\forall x Qx}$$

$$\forall x Px \wedge \forall x Qx$$

5-2'

$$\dfrac{\dfrac{\dfrac{\forall x Px \wedge \forall x Qx}{\forall x Px}}{Pa} \qquad \dfrac{\dfrac{\forall x Px \wedge \forall x Qx}{\forall x Qx}}{Qa}}{Pa \wedge Qa}$$

$$\forall x(Px \wedge Qx)$$

5-3

$$\dfrac{\forall x Px \vee \forall x Qx \qquad \dfrac{\dfrac{\forall x Px}{Pa}}{Pa \vee Qa} \qquad \dfrac{\dfrac{\forall x Qx}{Qa}}{Pa \vee Qa}}{Pa \vee Qa}$$

$$\forall x(Px \vee Qx)$$

(注) 5-3の逆の証明図（誤り）

$$\dfrac{\dfrac{\forall x(Px \vee Qx)}{Pa \vee Qa} \qquad \dfrac{\dfrac{[Pa]}{\forall x Px}}{\forall x Px \vee \forall x Qx} \qquad \dfrac{\dfrac{[Qa]}{\forall x Qx}}{\forall x Px \vee \forall x Qx}}{\forall x Px \vee \forall x Qx}$$

は，$\forall I$に関する条件のうち，

“ aが規則が適用される際に有効な仮定の中に含まれない定項でなくてはならない”

という条件に反しているため，逆は成り立たない！

5-4

$$\dfrac{\dfrac{\dfrac{\dfrac{[P] \quad \dfrac{\forall x\,(P \to Qx)}{P \to Qa}}{Qa}}{\forall x\, Qx}}{P \to \forall x\, Qx}}$$

5-5

$$\dfrac{\dfrac{\dfrac{\forall x\, Pax}{Pab} \quad \dfrac{\forall x \forall y\,(Pxy \to Qyx)}{Pab \to Qba}}{Qba}}{\forall x\, Qxa}$$

5-6

$$\dfrac{\dfrac{\dfrac{[\forall x\, Px] \quad \forall x\, Px \to \forall x\, Qx}{\forall x\, Qx}}{\dfrac{\neg Qa \quad Qa}{\times}}}{\neg \forall x\, Px}$$

6-1

$$\dfrac{\dfrac{\forall x\, Px}{Pa}}{\exists x\, Px}$$

6-2

$$\dfrac{\dfrac{\dfrac{\exists x(Px \wedge Qx) \quad \dfrac{[Pa \wedge Qa]^{①}}{Pa}}{\exists x\, Px}\,① \quad \dfrac{\exists x(Px \wedge Qx) \quad \dfrac{[Pa \wedge Qa]^{②}}{Qa}}{\exists x\, Qx}\,②}{\exists x\, Px \wedge \exists x\, Qx}$$

6-3

$$\dfrac{\dfrac{\dfrac{[Pa]^{①} \quad \dfrac{\forall x\,(Px \to Qx)}{Pa \to Qa}}{\dfrac{[\exists x\, Px]^{②} \quad \dfrac{Qa}{\exists x\, Qx}\,①}{\exists x\, Qx}}\,②}{\exists x\, Px \to \exists x\, Qx}$$

6-4

$$\dfrac{\dfrac{\dfrac{\dfrac{\exists x\,(Px \wedge Q) \quad \dfrac{[Pa \wedge Q]^{①}}{Pa}\,①}{Pa}}{\exists x\, Px} \quad \dfrac{\exists x\,(Px \wedge Q) \quad \dfrac{[Pa \wedge Q]^{②}}{Q}\,②}{Q}}{\exists x\, Px \wedge Q}$$

6-5

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\exists x \forall y Pxy}{\forall y Pay}}{Pab}}{\exists x Pxb}}{\forall y \exists x Pxy}}$$

6-6

$$\dfrac{\dfrac{\neg \exists x Px \qquad [Pa]}{\exists x Px}}{\dfrac{\dfrac{\times}{\neg Pa}}{\forall x \neg Px}}$$

6-7

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg Pa]^{①}}{\exists x \neg Px} \qquad [\neg \exists x \neg Px]^{②}}{\times}\,①}{\neg\neg Pa}}{Pa}}{\dfrac{\forall x Px \qquad \neg \forall x Px}{\times}\,②}$$

$$\dfrac{\neg\neg \exists x \neg Px}{\exists x \neg Px}$$

6-8

$$\dfrac{\dfrac{\dfrac{\dfrac{[Pa \wedge Qa]^{①}}{Qa} \qquad \dfrac{[Pa \wedge Qa]^{①}}{Pa}}{\neg Qa} \qquad \dfrac{\forall x (Px \to \neg Qx)}{Pa \to \neg Qa}}{\times}}{\dfrac{\times}{\neg \exists x (Px \wedge Qx)}\,②}\,[\exists x (Px \wedge Qx)]^{②}_{①}$$