

$$\textcircled{2} \tilde{A} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \dots & \dots & \dots \\ \vdots & \dots & \dots & \vdots \\ \tilde{a}_{n1} & \dots & \dots & \tilde{a}_{nn} \end{pmatrix} \quad \text{逆行列}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & \dots & \dots & \dots & \dots \\ \vdots & \dots & \dots & \dots & \vdots \\ a_{n1} & \dots & \dots & \dots & a_{nn} \end{pmatrix}$$

$$\begin{aligned} A\tilde{A} &= \begin{pmatrix} \sum_{i=1}^n a_{1i} \tilde{a}_{i1} & \sum_{i=1}^n a_{1i} \tilde{a}_{i2} & \dots & \sum_{i=1}^n a_{1i} \tilde{a}_{in} \\ \sum_{i=1}^n a_{2i} \tilde{a}_{i1} & \sum_{i=1}^n a_{2i} \tilde{a}_{i2} & \dots & \sum_{i=1}^n a_{2i} \tilde{a}_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^n a_{ni} \tilde{a}_{i1} & \sum_{i=1}^n a_{ni} \tilde{a}_{i2} & \dots & \sum_{i=1}^n a_{ni} \tilde{a}_{in} \end{pmatrix} \\ &= \begin{pmatrix} |A| & 0 & 0 & \dots & 0 \\ 0 & |A| & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & |A| \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \tilde{A}A &= \begin{pmatrix} \sum_{i=1}^n \tilde{a}_{i1} a_{i1} & \sum_{i=1}^n \tilde{a}_{i2} a_{i1} & \dots & \sum_{i=1}^n \tilde{a}_{in} a_{i1} \\ \sum_{i=1}^n \tilde{a}_{i1} a_{i2} & \sum_{i=1}^n \tilde{a}_{i2} a_{i2} & \dots & \sum_{i=1}^n \tilde{a}_{in} a_{i2} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^n \tilde{a}_{i1} a_{in} & \sum_{i=1}^n \tilde{a}_{i2} a_{in} & \dots & \sum_{i=1}^n \tilde{a}_{in} a_{in} \end{pmatrix} \\ &= \begin{pmatrix} |A| & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & |A| \end{pmatrix} = |A|E \end{aligned}$$