物理学基礎論 演習問

$\nabla\times(\nabla\phi)=0 \Longleftrightarrow \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\times\left(\frac{\partial\phi}{\partial x},\frac{\partial\phi}{\partial y},\frac{\partial\phi}{\partial z}\right)=0$ となる。

$\left(\frac{\partial}{\partial y}\frac{\partial\phi}{\partial z}-\frac{\partial}{\partial z}\frac{\partial\phi}{\partial y},\ \frac{\partial}{\partial z}\frac{\partial\phi}{\partial x}-\frac{\partial}{\partial x}\frac{\partial\phi}{\partial z},\ \frac{\partial}{\partial x}\frac{\partial\phi}{\partial y}-\frac{\partial}{\partial y}\frac{\partial\phi}{\partial x}\right)$

$=(0,0,0)=0$ よって(左辺)=(右辺)が成立 //

(左辺) $=\nabla\cdot(A\times B)=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$

$=\frac{\partial}{\partial x}(a_2b_3-a_3b_2)+\frac{\partial}{\partial y}(a_3b_1-a_1b_3)+\frac{\partial}{\partial z}(a_1b_2-a_2b_1)$ …①

(右辺) $=(b_1,b_2,b_3)\begin{pmatrix}\frac{\partial a_3}{\partial y}-\frac{\partial a_2}{\partial x}\\\frac{\partial a_1}{\partial x}-\frac{\partial a_3}{\partial x}\\\frac{\partial a_2}{\partial x}-\frac{\partial a_1}{\partial y}\end{pmatrix}-(a_1,a_2,a_3)\begin{pmatrix}\frac{\partial b_3}{\partial y}-\frac{\partial b_2}{\partial z}\\\frac{\partial b_1}{\partial z}-\frac{b_3}{\partial x}\\\frac{\partial b_2}{\partial x}-\frac{\partial b_1}{\partial y}\end{pmatrix}$

$=b_1\left(\frac{\partial}{\partial y}a_3-\frac{\partial}{\partial z}a_2\right)+b_2\left(\frac{\partial}{\partial z}a_1-\frac{\partial}{\partial x}a_3\right)+b_3\left(\frac{\partial}{\partial x}a_2-\frac{\partial}{\partial y}a_1\right)$

$-a_1\left(\frac{\partial}{\partial y}b_3-\frac{\partial}{\partial z}b_2\right)-a_2\left(\frac{\partial}{\partial z}b_1-\frac{\partial}{\partial x}b_3\right)-a_3\left(\frac{\partial}{\partial x}b_2-\frac{\partial}{\partial y}b_1\right)$ …②

ここで $\frac{\partial}{\partial x}(a_2b_3)=b_3\frac{\partial}{\partial x}a_2+a_2\frac{\partial}{\partial x}b_3$ より(他の文字、数字も同様)

② $=\frac{\partial}{\partial x}(a_2b_3-a_3b_2)+\frac{\partial}{\partial y}(a_3b_1-a_1b_3)+\frac{\partial}{\partial z}(a_1b_2-a_2b_1)$ …③

①.③より (左辺)=(右辺) //

$A=(a_1,a_2,a_3)$ とおくと

(左辺) $=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\begin{pmatrix}\frac{\partial a_3}{\partial y}-\frac{\partial a_2}{\partial z}\\\frac{\partial a_1}{\partial z}-\frac{\partial a_2}{\partial x}\\\frac{\partial a_2}{\partial x}-\frac{\partial a_3}{\partial y}\end{pmatrix}$

$=\frac{\partial}{\partial x}\frac{\partial a_3}{\partial y}-\frac{\partial}{\partial x}\frac{\partial a_2}{\partial z}+\frac{\partial}{\partial y}\frac{\partial a_1}{\partial z}-\frac{\partial}{\partial y}\frac{\partial a_3}{\partial x}+\frac{\partial}{\partial z}\frac{\partial a_2}{\partial x}$

$+\frac{\partial}{\partial z}\frac{\partial a_1}{\partial y}=0$ よって証明終 //