

I a) 單振動

b), c) 減衰振動

e) 強制振動

d) a) $\frac{1}{2} m \omega^2 x^2$

d) $\frac{1}{2} m \omega^2 x^2 + \frac{m \omega^2}{4A^2} x^4$

(2) a) $x(t) = A \sin \omega t$

b) $\ddot{x} = -\omega^2 x - \omega^2 x$

$x = A e^{w't}$ $\gamma < \omega$

$A(\omega'^2 + \omega \omega' + \omega^2) e^{w't} = 0$

$\omega' = \frac{-\omega \pm \sqrt{\omega^2 - 4\omega^2}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2} \omega$

$x = A e^{\frac{-1+\sqrt{3}i}{2} \omega t} + B e^{\frac{-1-\sqrt{3}i}{2} \omega t}$

$x(0) = A + B = 0$

$\therefore x = A' e^{-\frac{1}{2} \omega t} \sin \frac{\sqrt{3}}{2} \omega t$

$x'(t) = A' e^{-\frac{1}{2} \omega t} (-\frac{1}{2} \omega \sin \frac{\sqrt{3}}{2} \omega t + \frac{\sqrt{3}}{2} \omega \cos \frac{\sqrt{3}}{2} \omega t)$

$x'(0) = \frac{\sqrt{3}}{2} \omega A' = A \omega$

$\therefore x(t) = \frac{2A}{\sqrt{3}} e^{-\frac{1}{2} \omega t} \sin \frac{\sqrt{3}}{2} \omega t$

c) $x = A e^{w't}$ $\gamma < \omega$

$A(\omega'^2 + \omega \omega' + \omega^2) e^{w't} = 0$

$\omega' = -\omega$

$\therefore x(t) = A e^{-\omega t} + B t e^{-\omega t}$

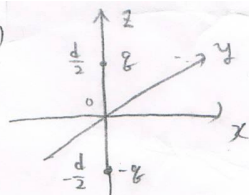
$x(0) = 0 \rightarrow A = 0$

$x'(t) = (B - B \omega t) e^{-\omega t}$

$x'(0) = B = A \omega$

$\therefore x(t) = A \omega t e^{-\omega t}$

II B)



$E_z(0,0,z) = \frac{1}{4\pi\epsilon_0} \left(\frac{g}{(z-\frac{d}{2})^2} + \frac{-g}{(z+\frac{d}{2})^2} \right)$

$= \frac{g}{4\pi\epsilon_0} \frac{1}{z^2} \left\{ \left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2} \right\}$

$\approx \frac{g}{4\pi\epsilon_0} \frac{1}{z^2} \left(1 + \frac{d}{z} - 1 + \frac{d}{z}\right)$

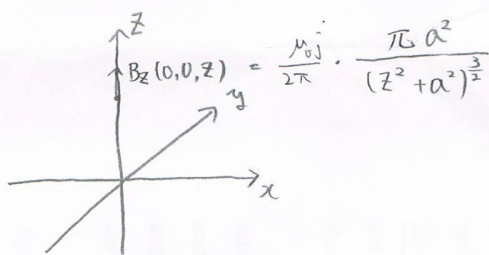
$= \frac{gd}{2\pi\epsilon_0} \frac{1}{z^3} = \frac{d}{z^3}$

$\therefore d = \frac{gd}{2\pi\epsilon_0}$

$\left. \begin{array}{l} gd \rightarrow m \\ \frac{1}{\epsilon_0} \rightarrow \gamma_0 \end{array} \right\}$

$\gamma = \frac{m \gamma_0}{2\pi}$

(4)



$B_z = \frac{\mu_0 j}{2\pi} \frac{\pi a^2}{z^2} \left(1 + \frac{a^2}{z^2}\right)^{-\frac{3}{2}}$

$\approx \frac{\mu_0 j}{2\pi} \frac{\pi a^2}{z^3} \left(1 - \frac{3a^2}{2z^2}\right) \quad z \gg a$

$\approx \frac{\mu_0 j}{2\pi} \frac{\pi a^2}{z^3}$

$= \frac{m \gamma_0}{2\pi} \frac{1}{z^3}$

$\therefore m = \pi a^2 j$